## Mathematical Reasoning Strand

Reasoning is an integral part of mathematics and requires several important skills, including examining patterns, making and testing conjectures, and using formal inductive and deductive reasoning to formulate mathematical arguments. Mastery of each of these elements of reasoning requires students to work with diverse problems and activities.

Activities in this strand require language with sufficient precision, clarity, and appropriateness to support rigorous thinking. Standards in mathematical reasoning require students to analyze problems by identifying relationships, to formulate and justify conjectures, to use estimation on the basis of numerical or graphical information, to use inductive and deductive reasoning, to evaluate the reasonableness of solutions, and to generalize results and apply them to new problems.

Each question in this strand is also classified within one of the four grades 6 and 7 mathematical strands for purposes of reporting student scores. None of the questions in this strand are classified in the Algebra I strand.

The six specific California academic content standards covered by the CAHSEE Mathematical Reasoning strand are discussed in the following pages.

Strand **Mathematical Reasoning (MR)**

Standard **7MR1.1**
**Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

**Chris drove 100 kilometers from San Francisco to Santa Cruz in 2 hours and 30 minutes. What computation will give Chris' average speed, in kilometers per hour?**

**A** Divide 100 by 2.5.

**B** Divide 100 by 2.3.

**C** Multiply 100 by 2.5.

**D** Multiply 100 by 2.3.

M03164

Students must be able to analyze situations to clarify a problem and to identify those elements that will make it possible to solve the problem. CAHSEE test questions for this standard emphasize the analysis of problems rather than their solutions. The components of the standard include the following: determining relationships, discriminating between relevant and irrelevant information, identifying missing information, sequencing and prioritizing information, and observing and identifying algebraic and geometric patterns.

*Sample Test Question*
The sample question requires students to understand the relationship between the given distance (100 km), the given time (2 hours 30 minutes), and the rate (unknown). The correct answer is choice A. Students must recognize that Chris' average speed is the unknown variable and must know how to use the distance, rate, time equation to determine the rate. This question is classified in the Measurement and Geometry strand for purposes of reporting student scores.

*Analysis of Distractors*
The distractors represent errors in the application of the given information and the equation. Distractor B indicates that division is the appropriate computation; however, it provides 2.3 as an incorrect value for 2 hours and 30 minutes. Distractor C presents an incorrect operation, multiplication, although the expression for the number of hours is correct. Distractor D presents multiplication as the operation and also presents an incorrect value for 2 hours and 30 minutes.

Strand **Mathematical Reasoning (MR)**

Standard **7MR1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.**

Constructs **Conceptual Understanding, Problem Solving**

**If $n$ is any odd number, which of the following is true about $n+1$?**

**A** It is an odd number.

**B** It is an even number.

**C** It is a prime number.

**D** It is the same number as $n-1$.

M00155

The challenge for many students in formulating mathematical conjectures is precision of language. CAHSEE test questions for this standard focus on both of its components: formulation of a conjecture and justification of a conjecture. Students may be asked to make conjectures based on indirect or incomplete evidence. Test questions may state a conjecture and ask students to choose among reasons that the conjecture is reasonable.

*Sample Test Question*

The sample question presents $n$ as any odd number and then requires evaluation of four conjectures. The correct answer is choice B. Students should reason that if $n$ is odd, $n+1$ is necessarily even. Students may also attempt to find counterexamples, i.e., examples of odd $n$ and also odd $n+1$. This question is classified in the Algebra and Functions strand for purposes of reporting student scores.

*Analysis of Distractors*

Students should reason that Distractor A is always false because $n+1$ must always be even. Distractor C is true only when $n=1$, since 2 is the only even prime. Distractor D is never true, since $n-1$ always differs from $n+1$ by 2.

Strand **Mathematical Reasoning (MR)**

Standard **7MR2.1**
**Use estimation to verify the reasonableness of calculated results.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

**The table below shows the number of visitors to a natural history museum during a 4-day period.**

| Day | Number of Visitors |
|---|---|
| Friday | 597 |
| Saturday | 1115 |
| Sunday | 1346 |
| Monday | 365 |

**Which expression would give the BEST estimate of the total number of visitors during this period?**

**A** $500 + 1100 + 1300 + 300$

**B** $600 + 1100 + 1300 + 300$

**C** $600 + 1100 + 1300 + 400$

**D** $600 + 1100 + 1400 + 400$

M11112

This standard emphasizes estimating and evaluating the reasonableness of results. This standard requires students to use estimating skills in computation and compare estimated results to calculated results in order to judge their reasonableness.

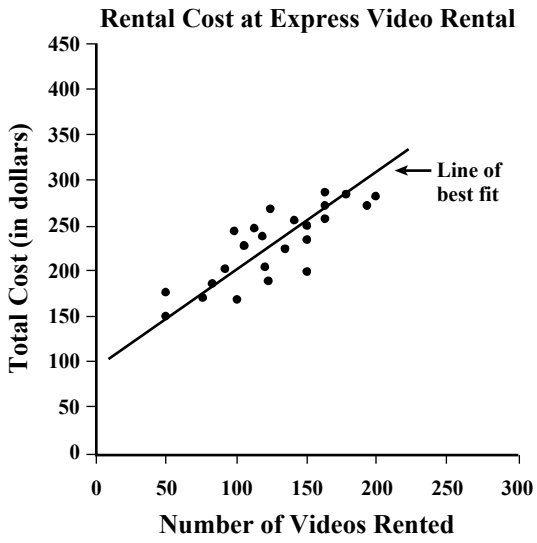
*Sample Test Question*
The sample question gives a table of 4 values and asks students to find the expression that will give the best estimate of the total of the values. The correct answer choice is C. Students should round each value to the nearest hundred. This question is classified in the Number Sense strand for purposes of reporting student scores.

*Analysis of Distractors*
The distractors represent rounding errors. Distractor A represents the error of rounding 597 to 500 and 365 to 300. Distractor B represents the error of rounding 365 to 300. Distractor D represents the error of rounding 1346 to 1400.

Strand **Mathematical Reasoning (MR)**

Standard **7MR2.3**
**Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**




**Using the line of best fit shown on the scatterplot above, which of the following best approximates the rental cost per video to rent 300 videos?**

**A** $3.00

**B** $2.50

**C** $2.00

**D** $1.50

M02209

Graphs provide a quick summary of data or of a relationship but may not include the specific information required to answer a particular question. By identifying trends and patterns and using interpolation and extrapolation, students may be able to obtain a reasonable estimate of the needed information. This content standard has two components: estimating graphically and solving for unknown quantities. CAHSEE test questions for the standard may ask students to find or identify the most accurate line of best fit through a scatterplot, to extract information from a graph by interpolation or extrapolation, or to identify an equation that could be used to solve a problem shown in a graph.

*Sample Test Question*
The sample question includes a scatterplot that relates total cost to number of videos rented. Finding the line of best fit requires an approximation of the correspondence between videos rented (*x*-axis) and total cost (*y*-axis). The correct answer is choice D. Students should recognize that extending the line to include an *x*-value that corresponds to 300 videos allows the interpretation of a corresponding value of the total cost on the *y*-axis. From the line of best fit, renting 300 videos corresponds to approximately $450, or $1.50 per video. This question is classified in the Statistics, Data Analysis, and Probability strand for purposes of reporting student scores.

*Analysis of Distractors*

Distractor A requires a total cost of approximately $900 to obtain a per video cost of $3.00, which is out of the range of the line of best fit. Distractor B requires a total cost of approximately $750 to obtain a per video cost of $2.50, also out of the range of the line of best fit. Similarly, Distractor C requires a total cost of approximately $600 to obtain a per video cost of $2.00, also out of the range of the line of best fit.

Strand **Mathematical Reasoning (MR)**

Standard **7MR2.4**
**Make and test conjectures by using both inductive and deductive reasoning.**

Constructs **Conceptual Understanding, Problem Solving**

**The table below shows values for *x* and corresponding values for *y*.**

| *x* | *y* |
|---|---|
| 21 | 3 |
| 14 | 2 |
| 28 | 4 |
| 7 | 1 |

**Which of the following represents the relationship between *x* and *y*?**

**A** $y = \frac{1}{7}x$

**B** $y = 7x$

**C** $y = x - 6$

**D** $y = x - 18$

M00377

Being able to identify patterns (inductive reasoning) and then testing the validity of the patterns (deductive reasoning) are key skills in many fields besides mathematics. This standard requires students to use inductive and deductive reasoning to make and test conjectures. CAHSEE test questions for this standard may require reasoning from general to specific, from specific to general, and by use of counterexample.

*Sample Test Question*
The sample test question requires students to use mathematical reasoning to determine the relationship between *x* and *y*. The correct answer is choice A. Students should be able to recognize that the values for *y* are $\frac{1}{7}$ that of the values for *x*. This question is classified in the Algebra and Functions strand for purposes of reporting student scores.

*Analysis of Distractors*
Distractor B represents the values of *x* being multiplied by 7. Distractor C is correct only when the value of *x* is 7. Distractor D is correct only when the value of *x* is 21.

Strand **Mathematical Reasoning (MR)**

Standard **7MR3.3**
**Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.**

Construct **Problem Solving**

**Len runs a mile in 8 minutes. At this rate how long will it take him to run a 26-mile marathon?**

**Which of the following problems can be solved using the same arithmetic operations that are used to solve the problem above?**

**A** Len runs 26 miles in 220 minutes. How long does it take him to run each mile?

**B** A librarian has 356 books to place on 18 shelves. Each shelf will contain the same number of books. How many books can the librarian place on each shelf?

**C** A cracker box weighs 200 grams. What is the weight of 100 boxes?

**D** Each basket of strawberries weighs 60 grams. How many baskets can be filled from 500 grams of strawberries?

M00137

One key problem-solving skill is to recognize how a new problem is like a simpler or more familiar problem. This content standard requires students to understand the process by which problems are reasoned, analyzed, and solved. CAHSEE test questions for this standard may include the selection of appropriate analogs to a given problem situation, and the relevance of the analogs as applied to the types of reasoning, patterns of operations, or logical extensions, rather than to context or other more superficial characteristics of the problem. Test questions for this standard may or may not require numerical solutions.

*Sample Test Question*
The sample test question presents a mathematical problem in context that requires multiplication to find a total. Students must select the most appropriate analog to finding the total time for running 26 miles at the rate of 1 mile per 8 minutes. The correct answer is choice C. Students should recognize the analog in using multiplication to determine the total weight of 100 cracker boxes, with 1 cracker box weighing 200 grams. This question is classified in the Number Sense strand for purposes of reporting student scores.

*Analysis of Distractors*
Distractor A requires the calculation of time (in minutes) per one mile, rather than per 26 miles as stated in the question. In Distractor B it is necessary to calculate the number of books per shelf by dividing 356 by 18. Distractor D is similar to B, since to find the number of 60-gram baskets in 500 grams, the appropriate step is to divide 500 by 60.